

# Spatial Acquisition of Optical Sources in the Presence of Intense Interference

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*The problem of spatially acquiring an optical beacon in the presence of intense optical interference is considered. It is demonstrated that conventional acquisition procedures cannot establish acceptable levels of acquisition probability when competing with strong optical interference, without requiring unreasonable amounts of prime power. As a possible solution, it is shown that a more sophisticated acquisition scheme can virtually eliminate the effects of optical interference, while requiring only a modest increase in system complexity.*

## I. Introduction

The problem of spatially acquiring an optical source in the presence of optical interference is considered. The spatial acquisition of the uplink beacon by a deep-space vehicle (DSV) is a necessary prerequisite to optical downlink transmission (Ref. 1). Similarly, a near-Earth optical relay receiver must acquire the downlink beam before it can decode the transmitted data, or relay it to the ground for decoding. Prior to spatial acquisition, the angular uncertainty in the location of the beacon tends to be considerable (on the order of  $10^{-8}$  steradians or more at the DSV). Consequently, the illuminated Earth, and possibly the Moon, might be included within the initial uncertainty region, along with the desired optical source. For the case of the relay receiver attempting to locate the DSV, the target planet (Jupiter, Saturn, etc.) is likely to be included in the initial uncertainty field-of-view. These extraneous optical sources interfere with spatial acquisition, and in extreme cases, may even cause the acquisition (and subsequent tracking) of the wrong source. In the following sections we

develop a model for the problem of acquiring the desired optical source (henceforth termed the "beacon") in the presence of a single interfering source. We shall demonstrate that while straightforward acquisition schemes invariably fail in the presence of intense interference, more sophisticated procedures can yield vast improvements in acquisition performance.

## II. The Spatial Acquisition Problem

The problem of acquiring an optical source in the presence of interference can be described by referring to Fig. 1. The angular uncertainty region is assumed to be encompassed by a square solid angle of  $\Omega_u$  steradians, roughly  $\theta_u$  degrees on a side. It is desired to resolve the angular position of the optical beacon to within  $\Omega_r$  steradians. Therefore, we can partition the uncertainty region into  $M = \Omega_u/\Omega_r$  resolution cells, each cell  $\theta_r$  degrees on a side, and search the resulting matrix of resolution cells to locate the beacon source.

The derivation of the acquisition probability is straightforward when both uniform background radiation and point-source interference are absent. Assuming the count-intensity of the beacon is  $n_s$  photoelectrons/second, the average number of photoelectrons generated in  $T$  seconds by a photodetector (whose active area corresponds to the dimensions of the appropriate resolution cell) is  $K_s = n_s T$ , while the average count generated over all other resolution cells is identically zero. Invoking the majority-count decision rule (Ref. 2), correct acquisition occurs if the optical source generates at least one count over the proper resolution cell. Assuming that the counts are Poisson distributed, the acquisition probability for this ideal case,  $PAC^*$ , becomes

$$PAC^* = 1 - \exp(-K_s) \quad (1)$$

This upper bound on the acquisition probability is shown in Fig. 2. Note that an average of at least 2.3 counts is required per observation interval in order to maintain better than 90 percent probability of acquisition, even in the absence of external interference.

Next, we consider the acquisition problem when a single interfering source is included in  $\Omega_u$ , along with the desired optical source. (The extension to more than one interfering source is straightforward, but computationally tedious.) The interference is assumed to be far enough removed from the desired source (and small enough in angular extent) to excite a different resolution cell, inducing a count-intensity of  $n_i$  photoelectrons/second, hence an average observed count of  $K_i = n_i T$  photoelectrons over that resolution cell. The resolution cell with the greatest number of observed counts is selected. Correct acquisition takes place if the beacon generates more counts than the interference: that is, if  $k_s > k_i$ . In case of a tie, the search is repeated (Ref. 2). The probability of correct acquisition,  $PAC$ , can therefore be expressed as

$$\begin{aligned} PAC &= \sum_{k_s=1}^{\infty} \frac{K_s^{k_s}}{k_s!} e^{-K_s} \left[ \sum_{k_i=0}^{k_s-1} \frac{K_i^{k_i}}{k_i!} e^{-K_i} \right] \\ &= \sum_{k_s=1}^{\infty} \frac{K_s^{k_s}}{k_s!} e^{-K_s} \left[ 1 - \frac{\gamma(k_s, K_i)}{(k_s-1)!} \right] \end{aligned} \quad (2)$$

where

$$\gamma(\alpha, x) \triangleq \int_0^x e^{-t} t^{\alpha-1} dt; \quad \operatorname{Re} \alpha > 0.$$

The exact computation of  $PAC$  is hindered by the infinite summation in Eq. (2). However, the truncated acquisition probability,  $PAC_T$ , can be computed for any finite summation limit  $\delta$ , where

$$PAC_T = \sum_{k_s=1}^{\delta} \frac{K_s^{k_s}}{k_s!} e^{-K_s} \left[ 1 - \frac{\gamma(k_s, K_i)}{(k_s-1)!} \right] \quad (3)$$

Defining the truncation error  $E_T$  as

$$E_T = \sum_{k_s=\delta+1}^{\infty} \frac{K_s^{k_s}}{k_s!} e^{-K_s} \left[ 1 - \frac{\gamma(k_s, K_i)}{(k_s-1)!} \right] \quad (4)$$

we observe that since  $E_T > 0$  for all finite  $\delta$ ,

$$PAC = PAC_T + E_T \geq PAC_T \quad (5)$$

for all  $\delta$ .

The truncated acquisition probability,  $PAC_T$ , therefore lower bounds the actual value of  $PAC$ . It is shown in the Appendix that  $E_T$  can be made negligibly small by letting  $\delta + 1 = \zeta K_s$ ,  $\zeta > 0$ , and selecting a sufficiently large value for  $\zeta$ . Having computed  $PAC_T$  under the above constraint, we shall henceforth let  $PAC_T \approx PAC$ .

Figure 2 shows the behavior of  $PAC$  as a function of  $K_s$  for increasing values of the average interference-induced count,  $K_i$ . It is apparent that weak interference ( $K_i \ll K_s$ ) has no appreciable effect on the acquisition probability. However,  $PAC$  deteriorates rapidly in the presence of strong interference ( $K_i \geq K_s$ ). One rather obvious solution to the interference problem is therefore to increase the source power until  $n_s$  becomes much greater than  $n_i$ , while holding the observation time constant. Unfortunately, this solution may not always be practical, due to the increased power consumption required.

If the beacon and interference count-intensities are comparable, but  $n_i < n_s$ , then  $PAC$  can be improved by increasing the observation interval, as shown in Fig. 3. Here we define  $\alpha$  as the ratio of interference-rate to beacon-rate,  $\alpha = n_i/n_s$ . When  $\alpha < 1.0$ ,  $PAC$  shows some improvement with increasing  $T$ , although the rate of improvement decreases as  $\alpha$  approaches unity. For  $\alpha = 1$ , the acquisition probability cannot be improved by increasing  $T$ , while for  $\alpha > 1$ , increasing the observation time leads to a catastrophic degradation in acquisition performance. This behavior is clearly illustrated by the curves corresponding to  $\alpha = 2$  and  $\alpha = 4$  in Fig. 3.

We have shown, therefore, that it becomes difficult to achieve high acquisition probabilities by increasing the source power or the observation time, in the presence of intense interference. Some additional advantage may be gained by a combination of the two techniques, that is, by increasing the source power to drive  $\alpha$  below unity, and simultaneously increasing the observation time. In the next section, we shall demonstrate a somewhat different acquisition technique that virtually eliminates the interference problem in acquisition systems, without incurring significant penalties in terms of system complexity.

### III. Spatial Acquisition of Pulsed Optical Sources

We have seen that strong optical interference can impede the spatial acquisition of the desired optical source. The difficulty is that with strong interference, the interfering counts  $k_i$  can, with high probability, exceed the beacon counts  $k_s$ , resulting in false acquisition. We propose to alleviate this problem by reducing the observation time in order to decrease  $K_i$ , while keeping  $K_s$  the same. This can be accomplished by pulsing the beacon laser and simultaneously transmitting the required timing information to the DSV over a separate RF channel. The requirement to transmit timing information to the DSV should not overburden the RF uplink, since optical acquisition is inherently a relatively short-duration operation.

We now assume that the beacon laser generates a periodic pulse-train, while maintaining constant average power. The received pulse-train can, therefore, be represented as in Fig. 4. The photoelectrons generated at the DSV by the beacon now appear concentrated in narrow, high count-intensity pulses of duration  $\tau'$ -seconds, at regular  $T'$ -second intervals. Defining the pulse-compression factor  $\gamma$  as  $\gamma = T'/\tau'$ , we observe that if the beacon count-rate is constrained to be  $\gamma n_s$  photoelectrons per second, then the average beacon count-rate remains  $n_s$  photoelectrons/second, thus maintaining constant average received power as required. As before, the interference count-rate remains  $n_i$  photoelectrons/second, uniformly distributed in time. The timing information can be used to define  $\tau$ -second observation windows around each  $\tau'$ -second pulse. Note that while the minimum value of  $\tau$  is  $\tau'$ , this limit can only be achieved with perfect timing. In general, the duration of the observation-window  $\tau$  would be much greater than  $\tau'$  in order to mask the effects of timing uncertainties. With little loss in generality, we shall argue that  $N$  pulses have to be observed in order to maintain the previously defined average signal count,  $K_s$ . Therefore, the average counts due to beacon and interference in the pulsed system become

$$K_s^p = N \gamma n_s \tau' = n_s T \quad (6a)$$

$$K_i^p = N n_i \tau \quad (6b)$$

We observe that the reduction in  $K_i^p$  over  $K_i$  depends directly on the ratio  $\tau/T'$ , which is, in turn, related to the accuracy of the timing information transmitted to the DSV. Defining the observation-compression factor  $\beta$  as  $\beta = T'/\tau$ , we can express the average interference counts in the pulsed system,  $K_i^p$ , in terms of  $K_i$ , as  $K_i^p = K_i/\beta$ . The interference-suppression capability of the pulsed system is, therefore, determined by the system-parameter  $\beta$ . (With present-day Q-switching technology and current DSN capabilities, maximum values of  $\beta$  on the order of a few hundred appear feasible.)

The improved performance of the pulsed system is clearly shown in Fig. 5, which shows the acquisition probability  $PAC$  as a function of  $\beta$  at the fixed value  $K_s = 5$ , for various  $K_i$  corresponding to high-intensity interference. (The upper-bound on acquisition probability,  $PAC^* = 0.9933$ , is also shown for comparison.) Figure 5 should be compared with Fig. 2 (at  $K_s = 5$ ) for a direct measure of the improvement provided by the pulsed system. Without pulsing (that is,  $\beta = 1$ ) the acquisition probability corresponding to  $K_s = 5$  and  $K_i = 10$  is  $PAC = 0.074$ , while even modest compression ratios ( $\beta > 20$ ) yield  $PAC \geq 0.98$ . With improved timing ( $\beta > 100$ ) reasonable acquisition probabilities can be obtained, even in the presence of extremely intense interference, as demonstrated by the curves corresponding to  $K_i = 20, 50$  and  $100$  in Fig. 5. Similar improvements over continuous intensity systems can be demonstrated at other values of  $K_s$  as well.

We have seen that in the presence of strong interference, acquisition probabilities can be improved either by increasing the average beacon power, or by pulsing the beacon (with a constant average power constraint) and relying on timing information transmitted in a separate channel to reduce the interfering counts. These techniques tend to increase system cost and complexity; therefore it may become desirable to employ a combination of both techniques in order to improve acquisition performance. Figure 6 shows graphs of constant  $PAC$  over the  $(\beta, K_s)$  plane. Each graph represents the locus of points corresponding to a constant acquisition probability in the presence of interference. It is apparent by a direct extension of these results that for any interference level  $K_i$ , a continuum of points can be found in the  $(\beta, K_s)$  plane to achieve any desired level of  $PAC$ . The two system parameters  $\beta$  and  $K_s$  can therefore be traded off on the basis of cost and complexity while maintaining the required level of acquisition performance.

## Appendix

The truncation error  $E_T$  defined in Eq. (4) can be upper bounded by  $E_T^u$ , as shown by the following inequality:

$$E_T = \sum_{k_s=\delta+1}^{\infty} \frac{K_s^{k_s}}{k_s!} e^{-K_s} \left[ 1 - \frac{\gamma(k_s, K_s)}{(k_s - 1)!} \right] \quad (7)$$

$$< \sum_{k_s=\delta+1}^{\infty} \frac{K_s^{k_s}}{k_s!} e^{-K_s} \triangleq E_T^u$$

The upper bound on the truncation error,  $E_T^u$ , can in turn be upper bounded by a direct application of the Chernoff bound (Ref. 3). This yields

$$E_T^u \leq e^{-\lambda(\delta+1)} e^{K_s(e^\lambda - 1)} \quad (8)$$

Equation (8) holds for any value of  $\lambda \geq 0$ . This inequality can be made exponentially tight by solving for that value of  $\lambda = \lambda_0$

that minimizes the right-hand side of Eq. (8). Differentiating with respect to  $\lambda$ , setting the result equal to zero and solving for  $\lambda_0$  yields

$$\lambda_0 = \ln \left( \frac{\delta + 1}{K_s} \right) \quad (9)$$

Substituting Eq. (9) into (7) and (8) and letting  $(\delta+1) = \xi K_s$ ,  $\xi > 0$ , we obtain

$$E_T < E_T^u \leq \xi^{-\xi K_s} e^{K_s(\xi-1)} = \left( \frac{e}{\xi} \right)^{\xi K_s} e^{-K_s} \quad (10)$$

This shows that for  $\xi > e$ ,  $E_T$  decreases at least exponentially with increasing  $K_s$ . In the computation of  $PAC_T$ , we have chosen the value  $\xi = 10$ . This implies that for any  $K_s \geq 1$ ,  $E_T < 8.103 \times 10^{-7}$ , and can therefore be neglected when compared to the range of acquisition probabilities under consideration.

## References

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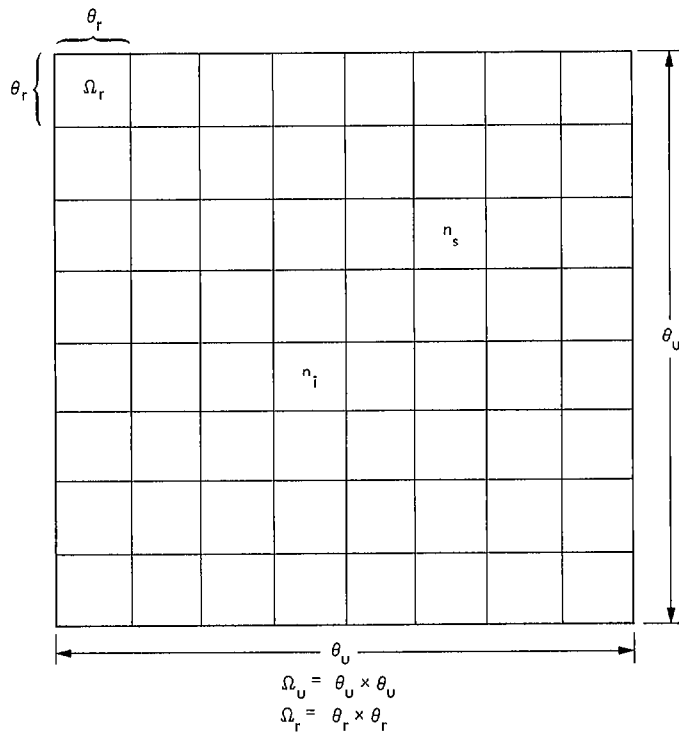


Fig. 1. Spatial resolution matrix

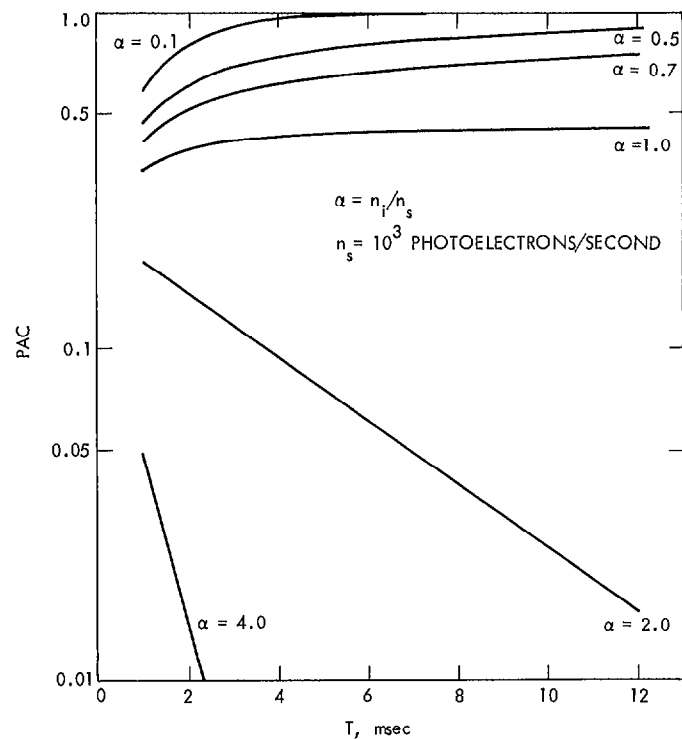


Fig. 3. Acquisition probability as a function of  $T$  for various  $\alpha$

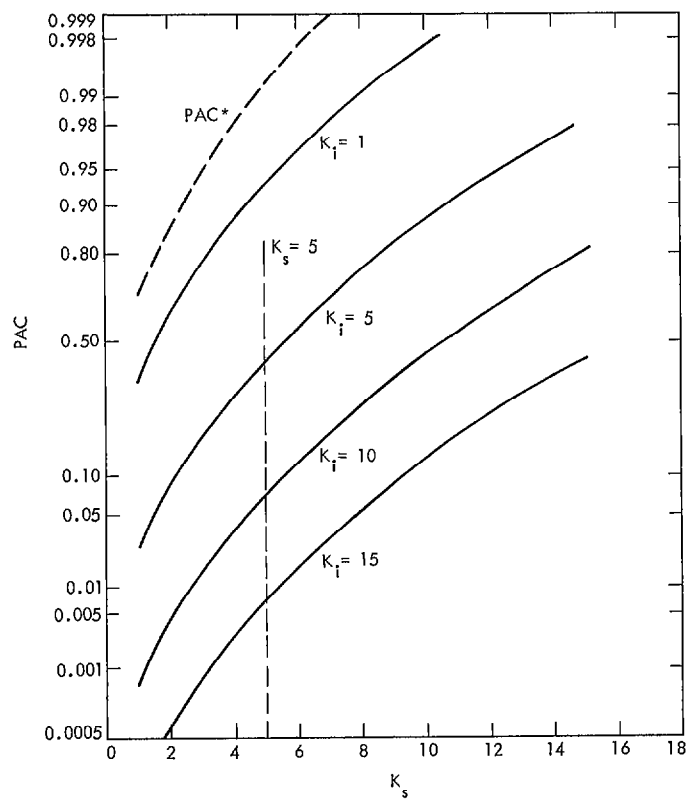


Fig. 2. Acquisition probability as a function of  $K_s$  for various  $K_i$

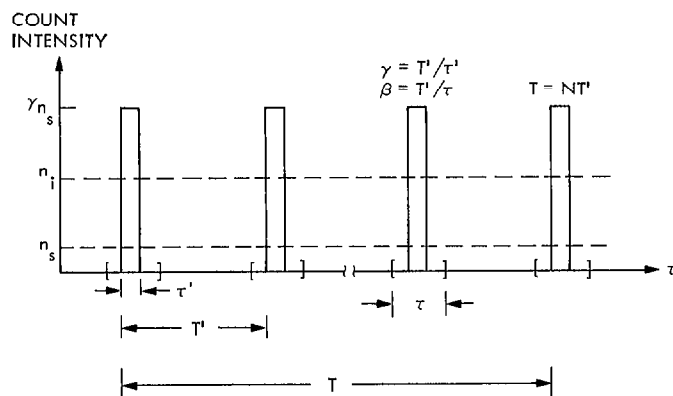


Fig. 4. Temporal distribution of count intensities for a pulsed beacon source

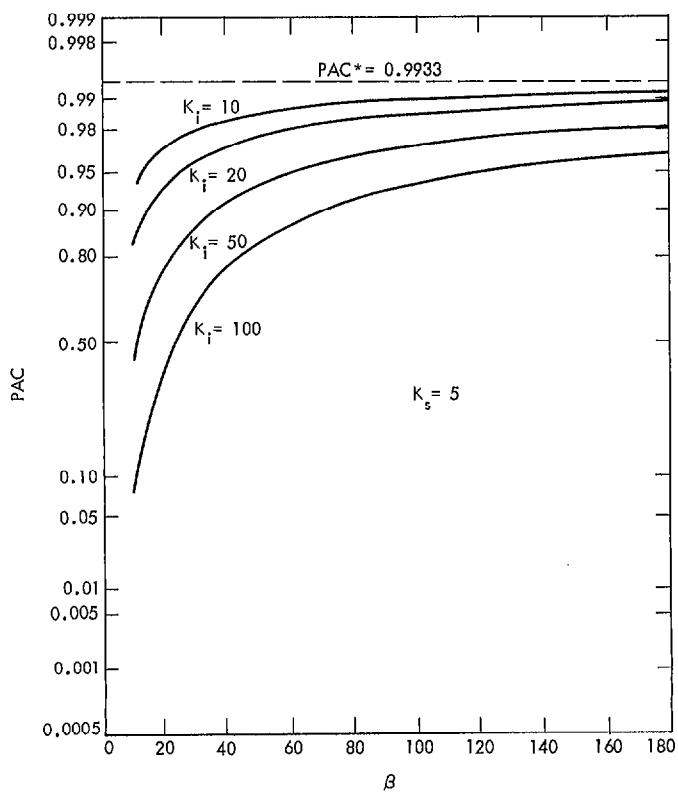


Fig. 5. Acquisition probability as a function of  $\beta$ ,  $K_s = 5$

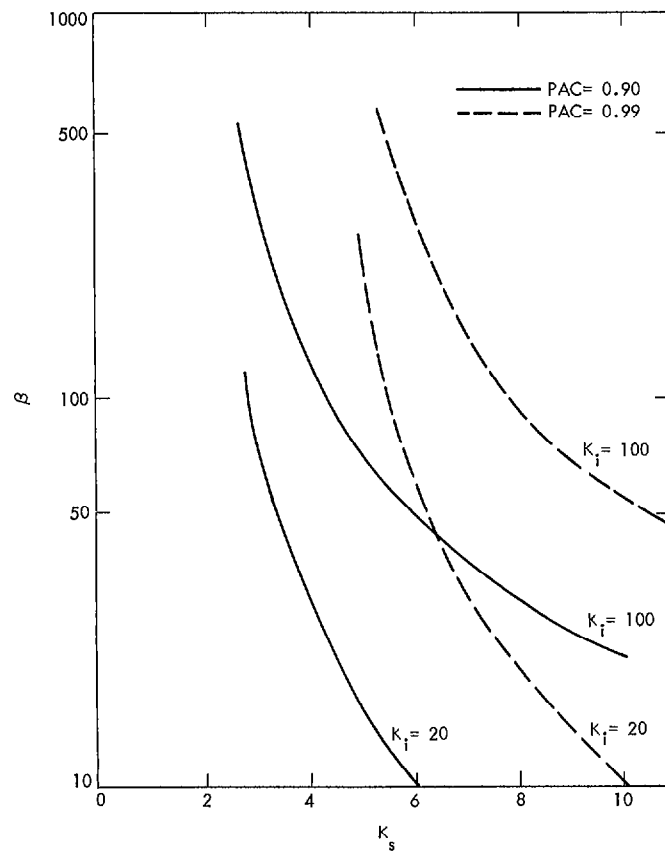


Fig. 6. PAC as a function of  $\beta$  and  $K_s$  for various  $K_i$